RASHTRASANT TUKADOJI MAHARAJ NAGPUR UNIVERSITY, NAGPUR

As per National Education Policy 2020

M.Sc. Mathematics

Curriculum Framework for Four Semester

Post Graduate Degree Course in Mathematics

M.Sc. Part I (Semester I and II)

With effect from
the Academic Year 2023-24

PROGRAM: M. Sc. Mathematics

Program Outcome:

PO1. Critical Thinking: *Take informed actions after identifying the assumptions that frame our thinking and actions, checking out the degree to which these assumptions are accurate and valid, and looking at our ideas and decisions (intellectual, organizational, and personal) from different perspectives.*

PO2. Problem Solving: *Solve problems from the disciplines of concern using the knowledge, skills and attitudes acquired from mathematics/ sciences/social sciences/humanities.*

PO3. Individual and Team Work: *Function effectively as an individual, and as a member or leader in diverse teams, and in wide variety of settings.*

PO4. Ethics: *Understand multiple value systems including your own, the moral dimensions of your decisions, and accept responsibility for them.*

PO5. Self-directed and life-long learning: *Demonstrate the ability to engage in independent and life-long learning in the broadest context socio-technological changes.*

PO6. Design/Development of Solutions: *Design solutions for complex science problems and design system components or processes that meet the specified needs with appropriate consideration for the public health and safety, and the cultural, societal, and environmental considerations.*

PO7. Computational Thinking: *Understand data-based reasoning through translation of data into abstract concepts using computing technology-based tools.*

PO8. Aesthetic Engagement: *Demonstrate and master the ability to engage with the arts and draw meaning and value from artistic expression that integrates the intuitive dimensions of participation in the arts with broader social, cultural and theoretical frameworks.*

Program Specific Outcome*:*

PSO1: ***Rational Thinking****: Students be able to formulate and develop Mathematical arguments in a logical manner to unravel the gist hidden in the problem at hand.*

PSO2: ***Problem solving ability****: Student should be able to think in a critical manner to process the data, and develop Mathematical problem-solving ability.*

PSO3: ***Revisiting the question****: Students should be able to recall basic facts, important milestones, discoveries in Mathematics and inculcate habit of rational thinking by which the problem at hand can be revisited, time and again, that helps in solving it.*

PSO4: ***Analytical ability****: In the growing field of research, it is necessary for students to learn to use some packages like Matlab, Scilab, Mathematica, Maxima, etc, so that*

analytical tools be available to investigate the functions, problems through graphs, programming, etc.

PSO5: ***Numerical Ability****: Using packages, students can make programs to solve some problems of which exact solutions are not available, using tools of Numerical analysis.*

PSO6: ***Simulation Ability****: The problems that cannot be solved directly, can at times be solved through techniques of simulation by students.*

PSO7: ***Research****: Students thus motivated would prepare themselves for research studies in Mathematics and related fields.*

PSO8: ***Application****: Student will be able to apply their skills and knowledge in Mathematics to various fields of studies including, science, engineering, commerce and management etc.*

M.Sc. Semester I (MATHEMATICS) ALGEBRA		
Sem I Paper - I DSC (Core) Code: MMT1T01	*Course Outcomes:* *CO1:* ***Foundational Knowledge****: Students will be able to update their basics of Group Theory, Discuss on various topic of group in algebra.* *CO2:* ***Elementary Skills****: Students will be able to understand the importance of Solvable and Nilpotent, Alternating groups.* *CO3:* ***Basic Analytic skills****: The main outcome of the course is to equip students with necessary basic analytic skills for problem solving on Sylow theorems.* *CO4:* ***Application****: By applying the principles of basic theorems of Algebra through the course curriculum, students can solve a variety of logical problems in science and engineering.*	Credit: 4 No. of hours 60

SYLLABUS: ALGEBRA

Unit I – Structure theorem of groups: Direct product of groups. Finitely generated abelian groups. Invariants of a finite abelian group. Sylow Theorems. Groups of order p^2 and pq.

Unit II - Unique factorization domains and Euclidean domains: Unique factorization domain. Principal Ideal domains. Euclidean domains. Polynomial rings over unique factorization domains.

Unit III – Normal and Separable Extensions: Irreducible polynomials and Eisenstein criterion. Adjunction of roots. Algebraic extensions. Algebraically closed fields. Splitting fields. Normal extensions. Multiple roots. Finite fields. Separable extensions.

Unit IV – Galois theory and its application: Automorphism groups, and fixed fields. Fundamental theorem of Galois theory. Fundamental theorem of algebra. Roots of unity and Cyclotomic polynomials. Cyclic extensions. Polynomials solvable by radicals. Ruler and compass constructions.

Reference Books:

1. Basic Abstract Algebra: Bhattacharya, Jain, and Nagpal, Second Edition, Cambridge University Press.
2. Topics in Algebra, I. N. Herstein, Second Edition, John Wiley. .
3. Abstract Algebra: David S.Dummit and Richard M. Foote, John Wiley.
4. Contemporary Abstract Algebra by J.A. Gallian, 4th Ed., Narosa, 1999.
5. Algebra by M. Artin, Prentice Hall Inc 1994.
6. Algebra, 3rd Edition by S. Lang, Addison-Wesley, 1999.

Suggested digital platform: NPTEL/SWAYAM/MOOCs

M.Sc. Semester I (MATHEMATICS) TOPOLOGY		
Sem I Paper - II DSC (Core) Code: MMT1T02	*Course Outcomes:* *CO1:* ***Foundational Knowledge:*** ***S****tudents will learn the basic concepts of topological space, metric spaces, product topology, closed sets, limit points and continuous function. Students will also get to know about interrelating these concepts with one another.* *CO2:* ***Elementary Skills****: Students will study about the connectedness of topological spaces. They will get to know about connectedness on real line with standard examples* *CO3:* ***Basic Analytic skills****: Students will study about covering spaces and relate it with compactness of the spaces. Students will gain analytical skill to relate compactness on real line, limit point compactness and local compactness.* *CO4:* ***Application****: Students will be able to think critically and apply the knowledge of topological spaces in the study of analysis and will be able to prove the standard results regarding countability and separation axioms.*	Credit: 4 No. of hours 60

SYLLABUS: TOPOLOGY

Unit I – Topological Spaces and Continuous functions – Topological spaces, Basis for a topology, the product topology on X x Y, subspace topology, closed sets and limit points, Continuous functions, Product topology, The metric topology.

Unit II - Connectedness: Connected spaces, connected subspaces of the Real line, Components and local connectedness.

Unit III – Compactness: Compact spaces, compact subspaces of the Real line, limit Point Compactness, Local Compactness.

Unit IV – Countability and separation axioms: The Countability axioms, The Separation axioms, Normal spaces, The Urysohn Lemma, The Urysohn Metrization Theorem, The Tietze Extension theorem

Reference Books:

1. Topology: J. R. Munkres, (second edition), Prentice Hall of India, 2002.
2. Foundations of General Topology: W. J. Pervin, Academic press, 1964.
3. Topology by Dugundji, Prentice Hall of India, New Delhi, 1975.
4. Introduction to Topology and Modern Analysis: G. F. Simmons, Mc Graw Hill 1963.
5. General Topology: J. L. Kelley, Van Nostrand, 1995.

6. Introduction to general Topology: K. D. Joshi, Wiley Eastern Ltd. 1983
7. Counter Examples in Topology by L. Steen and J. Subhash, Holt, Rinehart and Winston, New York, 1970.
8. General Topology by S. Willard, Addison - Wesley, Mass., 1970

Suggested digital platform: NPTEL/SWAYAM/MOOCs

M.Sc. Semester I (MATHEMATICS) **ORDINARY DIFFERENTIAL EQUATION**		
Sem I Paper - III DSC (Core) Code: MMT1T03	*Course Outcomes:* *CO1:* ***Foundational Knowledge****: Students will be able to study basic notions in Differential Equations and use the results in developing advanced mathematics.* *CO2:* ***Elementary Skills****: Students will able to solve problems modeled using linear differential equations having ordinary points and regular singular points and solve them by method of power series.* *CO3:* ***Basic Analytic skills****: The main outcome of the course is to equip students to develop techniques to solve differential equations that would help students sharpen their understanding of the Mathematical solutions with their characteristics.* *CO4:* ***Application****: By applying the principles of basic tools through the course curriculum, students can solve a variety of practical problems involving ordinary differential equations in science and engineering.*	Credit: 4 No. of hours 60

SYLLABUS: ORDINARY DIFFERENTIAL EQUATION (ODE)

Unit I – Linear Equations with variable coefficients: Initial value problems for the homogeneous equations. Solutions of the homogeneous equations, The Wronskian and linear independence, Reduction of the order of a homogeneous equation, The non-homogenous equations, Homogeneous equations with analytic coefficients, The Legendre equations.

Unit II - Linear Equations with regular singular points: The Euler equations, Second order equations with regular singular points, The Bessel equation, Regular singular points at infinity.

Unit III – Existence and uniqueness of solutions to first order equations: The method of successive approximations, The Lipschitz condition of the successive approximation. Convergence of the successive approximation, Approximations to solutions and uniqueness of solutions.

Unit IV - Existence and Uniqueness of Solutions to System of first order ODEs: An example- Central forces and planetary motion, Some special equations, Systems as vector equations, Existence and uniqueness of solutions to systems, Existence and uniqueness for linear systems, Green's function, Sturm Liouville theory.

Reference Books:

1) An introduction to ordinary differential equations by E. A. Coddington, (2012), Prentice Hall of India Pvt. Ltd. New Delhi.
2) Ordinary Differential equations by G. Birkoff and G. G. Rota, John Willey and Sons
3) Partial differential equations and boundary-value problems with applications by Mark Pinsky, AMS, 3rd edition (2011).
4) Differential Equations with Applications and Historical note by G. F. Simmons, McGraw Hill, Inc. New York. (1972)
5) Theory of ordinary differential equations by E. A. Coddington and Levinson, McGraw Hill, New York (1955)
6) Elementary differential equations by E. D. Rainvills, The Macmillan company, New York. (1964)

Suggested digital platform: NPTEL/SWAYAM/MOOCs

M.Sc. Semester I (MATHEMATICS) PRACTICAL - I **COMPUTATION WITH C /C++**		
Sem I Practical – I Code: MMT1P02	*Course Outcomes:* *Upon successful completion, students will have the knowledge and skills to:* *CO1. Execute C /C++ programs involving logical statements.* *CO2. Operate Mathematical operations and Logical operators in determining the general output of the problem.* *CO3. Determine roots of a cubic equation in general perspective.* *CO4. Understand in depth nuances of programming that would help them gain confidence and avail them job opportunities.*	Credit: 2 No. of hours 60

(Minimum **15** programs be executed using C /C++ programming in Math Lab)

List of topics for practical problems with C / C++ programming:

Write a C / C++ Program to:

1. Calculate area of a Circle, Surface area and volume of a sphere
   when its radius (integer value) is given (floating point number with two decimal places).
   π = 3.14 approx.
2. Check if a given Number is zero or positive or negative using if...else statement
3. Verify Wilson's theorem that a natural number $p > 1$ is a prime number if and only if
   $(p-1)! \equiv -1\ (mod\ p)$. Take p as an input.
4. Find the Largest and Smallest Number (integer) among Three Numbers (integers) using IF. . . Else statement and Logical operator
5. Find whether a given character is a Vowel or Consonant. A character is taken as input.
   The character may be in Upper Case or in Lower Case.
6. Calculate the Sum of First and the Last Digit of a given Number.
7. Verify Fermat's Little Theorem: If n is a prime number, then for every $a,\ 1 \leq a < n$,
   $a^{n-1} \equiv 1\ (mod\ n)$.
8. Count total number of digits in a given Integer (N)
9. Write a C program to find sum of following series where the value of N is taken as input
   1+ 1/2 + 1/3 + 1/4 + 1/5 + . . . 1/N
10. Check whether the given number N can be expressed as Power of 2 or not.
    For example 32 can be expressed as 2^5.
11. Print the following Pyramid pattern up to Nth row. Where N (number of rows to be printed) is taken as input. For example, when the value of N is 5 the pyramid will be printed as follows

```
*****
****
***
**
*
```

12. Find prime numbers between 1 and 200.
13. Find the Fibonacci Sequence 0, 1, 1, 2, 3, 5, 8, 13, 21, 34, 55, 89, 144, 233, ….
    and hence show ratio B/A converges to Golden ratio.
14. Read Two One Dimensional Arrays of same data type (integer type) and merge them
    into another One-Dimensional Array of same type.
15. Delete duplicate elements from an array of integers.
16. Print lower and Upper triangular matrices of a given square matrix.
17. Find roots of a quadratic equation, $a\ x^2 + b\ x + c = 0, a \neq 0$ with a, b, c as input.
18. Find roots of a cubic equation, $a\ x^3 + b\ x + c = 0, a \neq 0$ with a, b, c as input.
19. Find value of a determinant when 3×3 matrix is given as an input.

20. Find sum of all elements of each row of a matrix and trace of a diagonal matrix.

Reference Books:

1. Mathematical Algorithms:
   www. https://www.geeksforgeeks.org/mathematical-algorithms/

2. Let Us C : Authentic guide to C programming language (18th Edition) by Yashvant Kanetkar

3. Let Us C++ by Yashavant Kanetkar

M.Sc. Semester I (MATHEMATICS) (ELECTIVE – I) **INTEGRAL EQUATIONS** (Option – A)		
Sem I **Paper - IV** **DSE (Elective)** **Code: MMT1T04-A**	*Course Outcomes:* *CO1:* ***Foundational Knowledge****: The new concept of 'Integral Equations' will be introduced to students in which they will study different types of integral equations and various methods to solve them. Also, they will be taught integral transforms such as Hilbert transform.* *CO2:* ***Elementary Skills****: Students will be able to understand integral equations with different types of kernel and will be able to recognize their solving methods.* *CO3:* ***Basic Analytic skills****: The main outcome of the course is to teach student about integral equations and solving them using various transforms such as Laplace transform, Fourier transform, Hilbert transform, etc.* *CO4:* ***Application****: By applying the solving techniques, students can solve Fredholm Integral equations, Volterra Integral equations, Non-linear Integral equations and Integro-differential equations.*	**Credit 4** **No. of hours** 60

SYLLABUS: INTEGRAL EQUATIONS

Unit I – Integral equations: Preliminary concepts of integral equations. Some problems which give rise to integral equations. Conversion of ordinary differential equations into integral equations. Classification of linear integral equations. Integro-differential equations.

Unit II – Solution of Integral equations: Fredholm equations. Degenerate kernels. Hermitian and symmetric kernels. The Hilbert- Schmidt theorem. Hermitization and symmetrization of kernels. Solutions of integral equations with Green's function type kernels.

Unit III – Types of Integral equations: Types of Volterra equations. Resolvent kernel of Volterra equations, Convolution type kernels. Some miscellaneous types of Volterra equations. Non-linear Volterra equations. Fourier integral equations. Laplace integral equations.

Unit IV – Integral Transforms: Hilbert transform. Finite Hilbert transforms. Miscellaneous integral transforms. Approximate methods of solutions for linear integral equations. Approximate evaluation of Eigen values and Eigen functions.

Reference Books:

1) Integral Equations: A short course: L. G. Chambers: International text book company Ltd, 1976.W. Klingenberg (Springer).
2) Linear integral equation, Theory and techniques, Academic press, New York 1971.
3) Linear Integral Equation, Theory and Techniques by R.P. Kanwal, Academic Press, N.Y. (1971).
4) Linear Integral Equations by S.G. Mikhlin, Hindustan Book Agency, (1960).
5) A First Course in Integral Equations by A.M. Viazwaz, World Scientific (1997).
6) Integral Equation: A Short Course by L.I.G. Chambers, International Text Book Company Ltd. (1976).
7) Integral Transform for Engineers by Larry Andrews, Bhimsen Shiramoggo,, Prentice Hall of India (2003).
8) Integral equations and boundary value problems by M. D. Raisinghania, S. Chand publication.

Suggested digital platform: NPTEL/SWAYAM/MOOCs

M.Sc. Semester I (MATHEMATICS) (ELECTIVE – I) **FUZZY MATHEMATICS** (Option -B)		
Sem I Paper - IV DSE (Elective) Code: MMT1T04-B	*Course Outcomes:* *Upon successful completion, students will have the knowledge and skills to:* *CO4. Interpret problems involving uncertainty and its quantification.* *CO5. Understand fuzzy numbers and fuzzy arithmetic.* *CO6. Implement fuzzy logic in various problems involving uncertainty.* *CO4. Understand fuzzy systems and fuzzy control.*	Credit 4 No. of hours 60

SYLLABUS: FUZZY MATHEMATICS

Unit I: Fuzzy Sets:

Uncertainty, Imprecision and Vagueness, Fuzzy systems, Fuzzy Sets, Fuzzy Vs crisp set, Types of fuzzy sets, Operations on fuzzy sets, Extension principle of fuzzy sets.

Unit II: Fuzzy equations: Fuzzy numbers and arithmetic, Fuzzy equations, Lattice of fuzzy numbers, Fuzzy relations and fuzzy graphs, Fuzzy morphisms.

Unit III: Fuzzy Inference:
Fuzzy logic, Fuzzy connectives, Fuzzy inference, Fuzzy propositions, Fuzzy quantifiers, Inference from conditional fuzzy propositions.

Unit IV: Fuzzy Control:
Fuzzy systems and fuzzy control, Fuzzy rule-based system, Fuzzification and Defuzzification, Design of fuzzy controllers, Examples of fuzzy systems.

Reference Books:

1) Mathematics of Fuzzy Sets and Fuzzy Logic. Barnabas Bede, Springer.
2) Fuzzy Sets and Fuzzy Logic, theory and applications. George J. Klir and Bo Yuan, Prentice Hall India.
3) Timothy J. Ross, Fuzzy Logic with Engineering Applications (Third Edition), Wiley, 2010.
4) Henri Prade, Fuzzy Sets and Systems Theory and Applications: Didier Dubois, Academic Press,1980.

. Semester I (MATHEMATICS) **RESEARCH METHODOLOGY IN MATHEMATICS**		
Sem I **Paper - V** **Research Code: MMT1T05** (Mandatory)	Course Learning Outcomes: *Upon successful completion, students will have the knowledge and skills to:* *CO1. Recall and describe the fundamental concepts and principles of mathematics. Understand the research approaches and their significance in various fields and the different types of research designs and their characteristics.* *CO2. Apply research methods and approaches to investigate mathematical phenomena.* *CO3. Analyze the effectiveness and clarity of scientific communication and presentations.* *CO4. Describe the roles and dynamics within a group process, including teamwork and collaboration.* *CO5. Explain the concept of sponsored research and its implications for research ethics.* *CO6. Explain the basic principles of intellectual property rights (IPR) and their relevance in research*	**Credit 2** **No. of hours** 30

SYLLABUS: RESEARCH METHODOLOGY

Unit I: Research Process:
Introduction, Philosophy of Mathematics, Pure Mathematics, Applied Mathematics. The current state and Prospects of Geometry and Nonlinear

differential equations. Meaning, objective and motivation in research. Types of research. Research approaches and significance. Research process, criteria of good research, Challenges for research in India.
Defining research problem. Research design, Hypothesis: Formation – Techniques – Testing, Methods of theoretical research. Scientific communication, Presentations.

Unit II: **Research Project**:
Problem and project based learning, the group process. The project work process. Structure of Project report. Sponsored research, Ethics of research.
Intellectual Property Rights (IPR): Types of IPR: Patent, Copyright, Trade Mark, Design, Geographical Indication, Plant Varieties and Layout Design – Genetic Resource and Traditional Knowledge – Trade Secrets. IPR in India: Genesis and development. IPR in abroad - Major International Instruments concerning IPR: Paris Convention, 1883.

Reference Books:

1. Rama Nand Singh, *Research Methodology and Techniques in Mathematics*, Centrum Press,
   New Delhi, India.
2. C.R. Kothari, *Research Methodology*, New Age International (P)Ltd., India.
3. John Kuda, *Research Methodology*: A Project Guide for University Students,
   Samfunds Litterature.
4. B.L. Wadera, *Patents, trademarks, copyright, Designs and Geographical Judications*.
5. P. Narayanan (Eastern Law House), *Intellectual Property Law*.
6. Nithyananda, K V. (2019). *Intellectual Property Rights: Protection and Management in India*:
   Cengage Learning India Private Limited.
7. Neeraj, P., & Khusdeep, D. (2014). *Intellectual Property Rights in India*:
   PHI learning Private Limited.
8. Ahuja, V K. (2017). *Law relating to Intellectual Property Rights in India*:
   Lexis Nexis.
9. *Journal of Intellectual Property Rights (JIPR): NISCAIR*

M.Sc. Semester I: **PRACTICAL: II**

RESEARCH METHODOLOGY IN MATHEMATICS

Sem I Practical – II **Code: MMT1P02** (Mandatory)	**Course Outcomes:** *Upon successful completion, students will have the knowledge and skills to:* *CO1: Demonstrate installation and compilation of free Miktex software and Tex studio.* *CO2: Implement their knowledge of Latex in preparing Tex documents which can be converted into .pdf or .dvi files* *CO3: Prepare question papers of the examination* *CO4: Develop research article as per the learnings from research methodology.*	**Credit 2** **No. of hours** 60

Syllabus for Practical II

RESEARCH METHODOLOGY IN MATHEMATICS

(Note: **All** listed practical problems must be conducted in Mathematics Laboratory)

List of topics for practical problems:

Initially installation of Miktex, Tex Studio (or similar Latex software) and compilation of Tex document should be carried out.

1) Essay writing in Latex and developing its pdf
2) Writing 10 Mathematical formulas in Latex and its compilation
3) Preparing Resume in Latex for job prospects
4) Preparing question paper for examination
5) Beamer presentation on Intellectual Property Rights
6) Beamer presentation on Ethics of Research and research process
7) Latex document on Bibliography
8) Preparing Latex document with images
9) Preparing Latex document of research paper that includes section, subsection and bibliography
10) Preparing Latex document of research paper as per the requirement of the journal
11) Beamer presentation of Mathematical research paper

MSc Sem – II (Mathematics) Syllabus follows:

M.Sc. Semester II (MATHEMATICS) **REAL ANALYSIS**		
Sem II Paper - I DSC (Core) Code: MMT2T01	*Course Outcomes:* *CO1:* ***Foundational Knowledge****: Students will be able to update their basics knowledge in sequence, series, limit, continuity and differentiability.* *CO2:* ***Elementary Skills****: Students will be able to understand the importance of uniform convergence and topological manifold.* *CO3:* ***Basic Analytic skills****: The main outcome of the course is to equip students with necessary basic analytic skills for problem solving with functions of several variables.* *CO4:* ***Application****: By applying the principles of basic tools through the course curriculum, students can solve a variety of practical problems involving Manifold, sub-manifold and differentiable manifold.*	Credit 4 No. of hours 60

SYLLABUS: REAL ANALYSIS

Unit I – Sequence and Series of Function: Uniform convergence. Uniform convergence
and continuity. Uniform convergence and integration. Uniform convergence and differentiation. Equicontinuous families of functions. The Stone-Weierstrass theorem.
Algebra of functions.

Unit II – Functions of Several Variables: Differentiation. The Contraction Principle. The Inverse Function Theorem. The Implicit Function Theorem. The Rank Theorem. Partitions of unity.

Unit III – Introduction of Manifold and Functions of Mapping: The space of tangent vectors at a point of R^n. Another definition of Ta (R^n). Vector fields on open subsets of R^n. Topological manifolds. Differentiable manifolds. Real Projective space. Grassman manifolds. Differentiable functions and mappings.

Unit IV – Differentiable Manifolds and Submanifolds: Rank of a mapping. Immersion.
Sub manifolds. Lie groups. Examples of Lie groups. The action of a lie group on a manifold, Transformation groups. The action of a discrete group on a manifold, Covering manifold.

Reference Books:

1. Principles of Mathematical Analysis (Third Edition): Walter Rudin Mc GRAW – HILL Book Company.
2. An Introduction to Differentiable Manifolds and Riemannian Geometry: W. Boothby, Academic Press, 1975.
3. Methods of Real Analysis: R. R. Goldberg, John Wiley.
4. Introduction to Topological Manifolds (Second Edition): John M. Lee.
5. Mathematical Analysis by T. M. Apostal, Narosa.
6. Real and complex analysis by Walter Rudin.
7. Real analysis by Royden and Fitzpatrick.
8. Introduction to Smooth Manifolds by John M. Lee.
9. Structure and Geometry of Lie groups by Joachim Hilgert and Kari-Hermann Neeb

Suggested digital platform: NPTEL/SWAYAM/MOOC

M.Sc. Semester II (MATHEMATICS) **DIFFERENTIAL GEOMETRY**		
Sem II Paper - II DSC (Core) Code: MMT2T02	*Course Outcomes:* *CO1:* ***Foundational Knowledge****: Students will be introduced to the fundamentals of Differential Geometry primarily by focusing on the theory of curves and surfaces in three dimensional space.* *CO2:* ***Elementary Skills****: Students will be study about the curves and their global properties. Students will get to know about Geodesic curve and its existence conditions.* *CO3:* ***Basic Analytic skills****: Students will get the knowledge of fundamental quadratic forms of a surface, intrinsic and extrinsic geometry of surface, problem of Metrization and Triangulation.* *CO4:* ***Application****: By applying various definitions, theorems and formulas, students can solve different problems based on curved surfaces and their curvatures. It can be further used to analyse shapes and data on non-flat surfaces.*	Credit 4 No. of hours 60

SYLLABUS: DIFFERENTIAL GEOMETRY

Unit I – Definition of surface. Curves on a surface. Surfaces of revolution. Helicoids. Metric. Direction coefficients. Families of curves. Isometric correspondence. Intrinsic properties. Geodesics. Canonical geodesic equations.

Unit II - Normal property of geodesics. Existence theorems. Geodesic parallels. Geodesic curvature. Gauss Bonnet theorem. Gaussian curvature. Surfaces of constant curvature. Conformal mapping. Geodesic mapping.

Unit III – Second fundamental form. Principal curvatures. Lines of curvature. Developable. Developable associated with space curves. Developable associated with curves on surfaces. Minimal surfaces and ruled surfaces. Fundamental equations of Surface theory. Parallel surfaces.

Unit IV - Compact surfaces whose points are umbilics. Hilbert's lemma. Compact surfaces of constant Gaussian or mean curvature. Complete surfaces. Characterization of complete surfaces. Hilbert's theorem. Conjugate points on geodesics. Intrinsically defined surfaces. Triangulation. Two dimensional Riemannian manifolds. Problem of Metrization. Problem of continuation.

Reference Books:

1) An introduction to Differential Geometry by T. J. Wilmore; Oxford University Press.
2) A course in Differential Geometry by W. Klingenberg (Springer)
3) Geometry of curves and surfaces by do Carmo, Academic Press.
4) Riemannian Geometry and Tensor Calculus by Weatherburn C.,
   Schaum's Outline of trigonometry: Robert Moyer, Frank Ayres, 2012.
5) Differential Geometry a first course by D. Somasundaram, Narosa Publishing House, 2008.

Suggested digital platform: NPTEL/SWAYAM/MOOCs

M.Sc. Semester II (MATHEMATICS) **ADVANCE NUMERICAL METHODS**		
Sem II Paper - III DSC (Core) Code: MMT2T03	*Course Outcomes:* *CO1:* ***Foundational Knowledge: S****tudents will learn the basic methods and tools of numerical methods in root finding for linear and non-linear equations. They will learn about Newton's method, Muller's method and System of non-linear equations.* *CO2:* ***Elementary Skills****: Students will develop skills in analysing the methods of interpolation for a given data using polynomial interpolation, Newton's divided difference, forward differences and Hermite interpolation.* *CO3:* ***Basic Analytic skills****: Students will develop skills to approximate a function using appropriate theorems and numerical methods as a solution to the problems.* *CO4:* ***Application****: Students will be able to think critically to use Trapezoidal rule, Simpson's rule and Newton cotes integration formula for solving Mathematics modelling problems. They will be able to compare results of the problems by different methods.*	Credit 4 No. of hours 60

SYLLABUS: ADVANCE NUMERICAL METHODS

Unit I – Solution of Algebraic and Transcendental equations:
Absolute, relative and percentage errors. Method of False position, Rate of convergence of Regula-Falsi Method. Newton-Raphson Method for non-repeated real roots and for real multiple roots, and near multiple roots, Rate of convergence of Newton-Raphson formula.
Generalized Newton's method. Ramanujan's Method. Graffe's root-squaring method. Birge-Vieta Method. Lin-Bairstow Method for finding complex roots of a polynomial.

Unit II – Interpolation Theory: Finite differences: Forward, backward and central, Difference of a polynomial, Newton's formulae for interpolation, Central difference interpolation formulae: Gauss's, Stirling's, Bessel's, Everett's formula. Relation between Bessels' and Everett's formulae. Practical interpolation. Interpolation with unevenly spaced points: Lagrange's and Hermite's interpolation formula. Newton's general interpolation formula. Inverse interpolation. Method of successive approximation. Double interpolation.

Unit III – Least squares, Splines, Numerical Integration: Least square curve fitting procedures: Fitting a straight line, multiple linear least square, curve fitting by polynomials and sum of exponentials. Spline functions: Linear splines, Quadratic

splines, cubic splines. Numerical Integration: The Trapezoidal rule and Simpson's 1/3rd and 3/8th rule, Romberg Integration, Newton- Cotes integration formulae.

Unit IV – Numerical Solution of Differential Equations: Ordinary Differential Equations: Euler's method, Error estimates for Euler's method, Modified Euler's method, Runge-Kutta 2nd and 4th order methods. Predictor-Corrector methods: Adams-Moulton method, Milne's method, Simultaneous and higher order differential equations. Partial differential equations: Solution of Laplace's equation by Jacobi's method and Gauss-Seidel method, heat equation in one dimension.

Reference Books:

1. Finite Differences and Numerical Analysis by H. C. Saxena, S, Chand and Company Ltd, New Delhi.
2. Introductory methods of Numerical Analysis by S. S. Sastry, fifth edition, 2012, PHI Learning private limited, New Delhi.
3. An Introduction to Numerical Analysis by K. E. Atkinson, Johan Wiley and sons, Inc.
4. An introduction to numerical Methods and Analysis, by James F. Epperson
5. Schaum's Outline of Numerical Analysis by Francis Scheid.

Suggested digital platform: NPTEL/SWAYAM/MOOCs

MSc Semester- II: PRACTICAL: III **NUMERICAL SOLUTIONS WITH COMPUTER PROGRAMMING** (MATLAB / R PROGRAMMING / PYTHON, etc.)		
Sem II Practical - III Code: MMT2P03	**Course Outcomes:** *Students will able to:* *CO1: Learn about the application of numerical method.* *CO2: Understand Newton's method, Muller's method and solve System of linear and non-linear equations.* *CO3: Find the errors in the solution so obtained by various methods.* *CO4: Derive Numerical integration using Trapezoidal rule, Simpson's rule, Newton-Cotes formulae.* *CO5: Apply approximate numerical methods to solve the problems with more accuracy.* *CO6: Learn how to obtain solution of ordinary and partial differential equations numerically.* *CO7: Compare different methods in numerical analysis efficiently.*	Credit 2 No. of hours 60

SYLLABUS FOR PRACTICAL – III:
NUMERICAL SOLUTIONS WITH COMPUTER PROGRAMMING
(Minimum **15** programs be executed in Math Lab using one of the software MATLAB / R PROGRAMMING / PYTHON, etc.)

List of topics for practical problems:

Write a computer program to:

1. Find a real root of the equation $2x = log_{10}x + 7$ between 3 and 4 correct to 3 decimal places by regula-falsi method. Then generalize the program for any equation whose real root lie between a and b.
2. Find a real root of a cubic equation using Newton-Raphson method, correct to four decimal places.
3. Find a double root of the equation $x^3 - x^2 - x + 1 = 0$ by generalized Newton's formula.
4. Compare Newton-Raphson method and regula-falsi method for finding a root of the same equation in terms of rate of convergence.
5. Evaluate $\sqrt{12}$ by applying Newton's formula correct to three decimal places. Generalize the program.
6. Obtain cube root of positive integer N and verify it for 12 by Newton's formula.
7. Find the smallest root of the equation $x^3 - 9\,x^2 + 26x - 24 = 0$, by Ramanujan's method, generalize the program.
8. Develop Forward Difference Table 3.2 as suggested in Reference 1.
9. Find the cubic polynomial which takes the values: $y(1) = 24, y(3) = 120, y(5) = 336$ and $y(7) = 720$, and hence in particular find $y(8)$ by Newton's interpolation formula.
10. Fit a curve of the form $y = \frac{x}{a+b\,x}$ to the following data $(3, 7.148), (5, 10.231), (8, 13.509), (12, 16.434)$. Generalize the program with input of data set.
11. Fit a straight line of the form $Y = a_o + a_1\,x$ to the data (x_i, y_i):
$x_i = 1, \quad 2, \quad 3, \quad 4, \quad 5, \quad 6$
$y_i = 2.4, 3.1, 3.5, 4.2, 5.0, 6.0$
12. Evaluate $I = \int_0^1 \frac{1}{1+x^2}\,dx$ correct to 3 decimal places by Trapezoidal and Simpson $1/3^{rd}$ rule with $h = 0.5, 0.25, 0.125$ respectively.
13. Evaluate $I = \int_0^1 \frac{1}{1+x}\,dx$ correct to 3 decimal places by Romberg's method
14. Match $I = \int_0^1 \sqrt{1 - x^2}\,dx = \frac{\pi}{4} = 0.785398163$ (approx.) by Trapezoidal and Simpsons rules (both) with number of subintervals 10, 20, 30, 40, 50.
15. Determine the value of y when $x = 0.1$, given that $y(0) = 1$ and $y' = x^2 + y$ by Modified Euler's method.
16. Find $y(0.1)$ and $y(0.2)$ correct to 4 decimal places when $\frac{dy}{dx} = y - x$ and $y(0) = 2$ by 2^{nd} and 4^{th} order Runge-Kutta (R-K) methods, compare results.
17. Find $y(0.2)$ when initial value problem is given: $y' = 3x + \frac{y}{2}$; $y(0) = 1$ with h = 0.2, 0.1, 0.05 by Euler, Modified Euler and 4^{th} Order R-K method. Compare the results.
18. Find $y(0.8)$ and $y(1.0)$ by solving $y' = 1 + y^2$ with $y(0) = 0$ by 4^{th} order R-K method and correct these values by Milne's method.
19. Solve numerically the equation $y' = y + x$ with initial condition $y(0) = 1$ by Milne's method from $x = 0$ to $x = 0.4$ (Refer book by H. C. Saxena, Reference 2)
20. Solve $y' = xy$ for $x = 1.4$ when initially $y(1) = 2$ by 4^{th} order R-K method.

Reference Books:

1. Introductory methods of Numerical Analysis by S. S. Sastry, fifth edition, 2012, PHI Learning private limited, New Delhi.
2. Finite Differences and Numerical Analysis by H. C. Saxena, S. Chand and Company Ltd, New Delhi.
3. An Introduction to Numerical Analysis by K. E. Atkinson, Johan Wiley and sons, Inc.
4. An introduction to numerical Methods and Analysis, by James F. Epperson Schaum's Outline of Numerical Analysis by Francis Scheid.

M.Sc. Semester II (MATHEMATICS) (Elective-II) **CLASSICAL MECHANICS**		
Sem II Paper - IV DSE (Elective-II) Code: MMT2T04-A	*Course Outcomes:* *CO1:* ***Foundational Knowledge****: Students will be able to update their basics of variational principle.* *CO2:* ***Elementary Skills****: Students will be able to understand the importance of Lagrange's equation of motion.* *CO3:* ***Basic Analytic skills****: The main outcome of the course is to equip students with necessary basic analytic skills for problem solving using Lagrange's and Hamilton's equations of motion.* *CO4:* ***Application****: By applying the course curriculum, students can solve a variety of practical problems in research.*	Credit 4 No. of hours 60

SYLLABUS: CLASSICAL MECHANICS

Unit I: Variational principle and Lagrange's Equations: Hamilton's principle, some techniques of the calculus of variations. Derivation of Lagrange's Equations from Hamilton's Principle. Extension of principle to nonholonomic systems. Conservation theorems and symmetry properties.

Unit II: Hamilton's Equations of motion: Legendre transformations and the Hamilton equations of motion, cyclic coordinates and conservation theorems, Routh's equations, Derivation of Hamilton's equations from a variational principle, the principle of least action.

Unit III: Canonical transformations: The equations of Canonical transformation, examples of canonical transformations. Symmetric approach to Canonical Transformation, Poisson's bracket and other canonical invariants.

Unit IV: Hamilton-Jacobi theory: Equations of motion. Infinitesimal canonical transformations and conservation theorems in the Poisson bracket formulation,

the angular momentum poisson bracket relations. Hamilton-Jacobi theory for Hamilton's principle, and Hamilton-Jacobi theory for characteristic functions.

References:

1. Classical Mechanics, H. Goldstein, Second edition, Narosa Publishing House, New Delhi
2. Dynamics Part-II, A. S. Ramsey, the English Language Book Society and Cambridge University Press.
3. Classical Mechanics, Gupta, Kumar and Sharma
4. Classical Mechanics, N. C. Rana & P. S. Joag , Tata Mc Graw Hill
5. Classical Mechanics, L. M. Katkar, Shivaji University Kolhapur, 2007

M.Sc. Semester II (MATHEMATICS) (Elective-II) **OPERATION RESEARCH**		
Sem II Paper - IV DSE (Elective-II) Code: MMT2T04-B	*Course Outcomes:* *CO1:* ***Foundational Knowledge****: Students will be able to update their basics of computational procedures of Linear Programming Problem.* *CO2:* ***Elementary Skills****: Students will be able to understand the importance of efficient computational procedures. Revised simplex method is a modification of the simplex method and students would know that it is economical on computer as it computes only relevant information.* *CO3:* ***Basic Analytic skills****: The main outcome of the course is to equip students with necessary basic analytic skills for problem solving using a modified computational procedure.* *CO4:* ***Application****: By applying the Revised simplex method and Network techniques through the course curriculum, students can solve a variety of practical problems in business, research and development, production & investment Marketing and engineering.*	Credit 4 No. of hours 60

SYLLABUS: OPERATION RESEARCH

Unit I – <u>Linear Programming Problem-Advanced Techniques</u>: Simplex Method, Revised Simplex Method (with and without artificial variables). Post optimality Analysis by changes in (i) objective function, (ii) requirement vector, (iii) coefficient matrix. Addition and deletion of variables, addition of constraints.

Unit II - <u>Integer programming</u>: Pure and mixed integer programming problem. Gomory's cutting plane algorithm. Fractional cut method-All integer L. P. P. and

mixed integer L. P. P. algorithms. Branch and Bound method.

Unit III – <u>Bounded variables in LPP</u>: Bounded variable techniques for L. P. P., unconstrained optimization. Constrained optimization with equality constraints-Lagrange's multiplier method. Interpretation of Lagrange multiplier. Constriained optimization with inequality constrained-Kuhn-Tucker conditions.

Unit IV - <u>Network Scheduling by PERT/CPM</u>: Network flow problems. Minimal spanning Tree problem. Shortest Route problems. Network basic components. Logical sequencing. critical path analysis. Program Evaluation and Review technique (PERT) and Critical Path Method (CPM).

<u>Reference Books:</u>

1) Operations Research: Kanti Swarup, P. K. Gupta and Man Mohan: S. Chand and Sons, New Delhi

2) Operation Research: Theory and Applications, by J. K. Sharma, Macmillan, 1997.

3) Introduction to Operations Research, by F. S. Hillier, G. J. Lieberman, McGraw-Hill, 2001

4) Operations Research: Theory, Methods and Applications, by S. D. Sharma, H. Sharma, Kedar Nath, Ram Nath, 1972

Suggested digital platform: NPTEL/SWAYAM/MOOCs

M.Sc. Semester II (MATHEMATICS) PRACTICAL - IV **ON JOB TRAINING / FIELD PROJECT**		
Sem II Practical -IV Code: MMT2P04	*Course Outcomes:* *On completion of course, Students will be able to:* *CO1: Acquire hands on training* *CO2: Know different aspects of the Institute/Industry involved in it* *CO3: Learn how to work in Team set up* *CO4: Develop aspiration to work up the ladder in the Institute/ Industry*	Credit 4 No. of hours 120

INSTRUCTIONS FOR ON JOB TRAINING / FIELD PROJECT

120 h (8 h per week) *100 Marks*

On job training or a Field Project is a skill based practical programme. This program can be carried out in two ways:

1. Training in external research Institute/ National Institute/ industry/ company based on mathematical applications. This program can be carried out with one External Mentor from the sponsoring institute and Internal Mentor from the Department of Mathematics of the College. The student has to undergo training of 120 hrs during M.Sc. Sem-II programme. The work carried out has to be submitted to the Head of the Department in the form of Project Report duly signed by the External Mentor and Internal Mentor. Continuous Internal Evaluation (CIE: 50 marks) will be assigned jointly by the two mentors while Semester End Examination (SEE: 50 marks) will be based on presentation of the work and viva by External Examiner appointed by university.

2. A field-based project can be assigned by the Internal Mentor from the Department only. However, such project will be based on field activity that will lead to skill enhancement. The work carried out has to be submitted to the HOD of the Institute/College in the form of Project Report duly signed by the Internal Mentor. Continuous Internal Evaluation (CIE: 50 marks) will be assigned by the Internal Mentor while Semester End Examination (SEE; 50 marks) will be based on presentation of the work and viva by External Examiner appointed by the University.